Solution of MHD Flow of Fluid and Heat Transfer Problem via Optimal Homotopy Asymptotic Method

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Abstract: In this paper, we use the optimal homotopy asymptotic method (OHAM) for the solution of Non-linear two dimensional incompressible laminar boundary layer flow of fluid in the presence of transverse magnetic field. The obtained results for the stream function and velocity profile were comparable in terms of accuracy with that obtained by A. Abdullah (2009) who studied the heat transfer problem using homotopy analytic method and compared with the numerical results reported previously.

Key Words: - Porous plate, MHD Flow, Magnetic Parameter, Dimensionless variable & Laminar Flow.

I. INTRODUCTION

The study of MHD flow and heat transfer over a stretching sheet is one of the very important problems in fluid mechanics. The problem of an electrically conducting fluid over a stretching porous plate in a porous medium with an external transverse uniform magnetic field has many applications in petroleum industry; purification of crude oil and fluid droplets sprays wire and fiber coating and polymer technology, production of plastic sheets and other instrument. All these processes depend on the physical properties of the fluid around the sheet. To understand the features of the flow over stretching sheet, various studies have been done traditionally for Newtonian fluids. S.D. Nigam and S.N. Singh [1], have studied the heat transfer by laminar flow between parallel plates under the action of transverse magnetic field. The results have obtained for heat transfer problem corresponding to Hartmann's velocity profile for forced flow between two infinite parallel plates. A simplified case valid for large peclet numbers has been worked out numerically. The mean mixed temperature and local total Nusselt number are tabulated and shown graphically. These results are compared with the corresponding values for the heat transfer problem in which the magnetic field is absent and the fluid is electrically non-conducting. They found that due to ionic-conductivity the mean missed temperature at any point is decreased and consequently the local total Nusselt number is increased. K. Vajravelu and J. Nayfeh [2], have studied the Hydromagnetic flow of dusty fluid over a stretching sheet. They have reduced the equation of motion in coupled non-linear ordinary differential equations. These coupled non-linear ordinary differentials are solved numerically on a IBM 4381with double precession, using a variable order, variable step size finite difference method. The numerical solutions are compared with their approximate solution, obtained by perturbation technique. For small values of β the exact (numerical) solution is in close agreement with that of the analytical (approximate) solution. It is observed that, even in the presence of a transverse magnetic field and suction, the transverse velocity of both fluid and particle G phases decreases with an increase in the fluid particle interaction parameter, β, or the particle-loading parameter, k.

Moreover, the particle density maximum at the surface of the stretching sheet, and the shearing stress increases with an increase in β or k. In engineering and scientific problems as flow of fluid and heat transfer from different geometries are usually non-linear and only a small number of these problems can be solved exactly. Some nonlinear equations are normally solved by using numerical techniques. Shijun Liao [3], has studied the homotopy analysis method for nonlinear problems and results obtained by this analysis agrees well with numerical results and can be regarded a definition of the solution of the considered nonlinear problem.

Hang Xu and S.J. Liao [4], have studied the homotopy analytic solution for the unsteady power law fluid flows on an impulsively stretching sheet taking into consideration the hall effects. They obtained a convergent series solution by HAM method. T. Hayat et al. [5], has studied homotopy solution for the channel flow of a third grade fluid. They have compared the solutions with exact numerical solution for various values of the physical parameter and found that a proper choice of the auxiliary parameter occurring in HAM solution generate very close results.

M. Sajid and T. Hayat [6], have studied non-similar series solution for boundary layer flow of a third-order fluid over a stretching sheet. They obtained the solution of nonlinear ordinary differential equation by homotopy analysis method (HAM), and also compared with the existing results in the literature. Rafael Cortell [7], has studied MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species. They have obtained the variations of dimensionless surface concentration and dimensionless surface concentration gradient as well as mass transfer characteristics with various parameters and Graphs has also been plotted. Their numerical computations show that the effect of destructive chemical reaction is to diminish the concentration boundary layer.

Recently some researchers have worked out on the boundary layer problem of flow of Non-Newtonian fluid over stretching sheet. Z. Zang and J. Wang [8], have studied the similarity solution of MHD flow of power-law fluids over a stretching sheet. They obtained a rigorous mathematical analysis for a MHD boundary layer problem, which arises in the two dimensional steady laminar boundary layer flows for an incompressible electrically conducting power law fluid along a stretching flat sheet in the presence of an exterior magnetic field orthogonal to the flow. In the self-similar case, the problem is transformed into a third order nonlinear ordinary differential equation with certain boundary conditions, which is proved to be equivalent to a singular initial value problem for an integro-differential equations of first order. Hang Xu and S. J. Liao [9], have studied dual solution of boundary layer flow over an upstream moving plate. They have obtained the dual solutions in series expression with the proposed technique, which agree well with numerical results. This indicates that the homotopy analysis method is an open system, in the framework of this technique.

S. Awang kechil and I. Hashim [10], have studied the steady two–dimensional laminar MHD Hiemenz flow against a flat plate with variable wall temperature in a porous medium which was solved numerically using the implicit finite difference technique of keller-box method. They obtained an approximate analytical solution of the problem by a simple analytical solution of the adomian decomposition method (ADM). A. Abdullah [11], has studied homotopy analytical solution of MHD fluid flow and heat transfer problem, and obtained analytical uniformly valid solution. The solutions has been verified graphically as well as numerically and compared with the numerical results reported previously.

Sheikholeslami et al. [12], has studied the Analytical investigation of Jeffery-Hamel flow with high magnetic field and nano particle by Adomian decomposition method, the traditional Navier-stokes equation of fluid mechanics and Maxwell's electromagnetism governing equations are reduced to nonlinear ordinary differential equations to model the problem and obtained results are well agreed with that the Runge-Kutta method. Sheikholeslami M and D.D. Ganji [13], have studied the Heat transfer of Cu-water nano fluid flow between parallel plates analytically using homotopy perturbations method. They have obtained the relation between Nusselt number and nano particle volume fraction. Mabood et al. [14], have studied the Blasius equation and solved it using a new technique called the optical homotopy asymptotic method. The solution procedure was found to be effective and has advantages in comparison to HAM. They also provided an approximate solution without any assumption or linearization.

S. Zuhra et al. [15], have studied the singular boundary value problems by optimal homotopy asymptotic method, the solutions have been compared with the solutions of another method named as modified adomian decomposition (MADM). For testing the success of OHAM, both of the techniques have been analyzed against the exact solutions in all problems. It has been found that the OHAM solution converges rapidly. Bewar A. Mahmood et al. [16], have studied homotopy analysis method for solving nonlinear diffusion equation with convection term, and compared the results with exact solution. S. Islam et al. [17], has studied Application of Optimal homotopy method of Benjamin-Bona-Mahony and Sawada-Kotera Equations. They compared the results with Homotopy perturbation method and exact differential equation. OHAM is found efficient and fast convergent during computation by Mathematica. Bathayna S. Kashkari et al. [18], have studied the application of Optical homotopy asymptotic method for the approximate solution of kawahara equation. They compared the obtained results with exact solution, homotopy perturbation method, variation homotopy perturbation method and variation interation method.

Hakeem ullah et al. [19], has studied the formulation and application of optical homotopy asymptotic method to coupled differential difference equation. To see the efficiency and reliability of the method, they consider relativistic Toda coupled nonlinear differential-difference equation, and provided a convenient way to control the convergence of approximate solutions when it is compared with other methods of solution found in the literature. N. R. Anakira et al. [20], have studied multistage optimal homotopy asymptotic method for solving initial value problems. The main advantage of this study is to obtain continuous approximate analytical solutions for a long time span. Numerical examples are tested to highlight the important features of the new algorithm, comparison of the MOHAM results, standard OHAM, exact solution and fourth-order Runge Kutta method.

In this present paper, our objective is to implement the OHAM method to MHD flow of fluid and heat transfer problem. The solution expresses auxiliary parameter p and k, which is then varied to determined its optimum value. For various values of p, k and magnetic parameter M, the solutions have been obtained in this present paper.

II. MATHEMATICAL FORMULATION

A: Basic Problem Formulation of Non-Linear equation:-

We know that the MHD flow of viscous, incompressible, electrically conducting fluid on a linear stretching surface and heat transfer, in the presence of a transverse constant magnetic field and a uniform free stream of constant velocity and temperature governed by the continuity equations, the momentum equation and the energy equation, the boundary conditions read, respectively as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 y}{\partial y^2}+g\beta T-\frac{\sigma\beta_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho\, C_p}\,\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

And the boundary conditions are

$$u(x,0)=ax\,,\ v(x,0)=0,\ T(x,0)=T_w=const, \tag{4}$$

$$u(x,\infty)=0\ \ and\ \ \ T(x,\infty)=0. \tag{5}$$

Here a is the stretching rate constant, u, v are the velocity of fluid, υ is the kinematic viscosity, g is the acceleration due to gravity, β is the coefficient of thermal expansion, T is the temperature, σ is the electric conductivity, B_0 is the magnetic field, ρ is the density and k is the thermal conductivity of the laminar fluid. In terms of the stream function the velocity components are

$$u=\frac{\partial\psi}{\partial y}\,,\quad v=-\frac{\partial\psi}{\partial x}. \tag{6}$$

Now in the above equation using the transformation

$$\psi=x\sqrt{a\,\upsilon}\,F(\eta),\ \ \eta=y\sqrt{\frac{a}{\upsilon}}\ \ ,\ \ \ \theta=\frac{T}{T_w}. \tag{7}$$

Now the system of equation of (1), (2), (3) we get the transformation

$$F'''+F\,F''-F'^2+G_r\,\theta-M^2F'=0, \tag{8}$$

$$\theta''+P_r\,F\,\theta'=0\,, \tag{9}$$

here the differentiation denotes with respect to , and F, θ is only the function of η, $G_r = g\beta\, T_w/(a^2 x)$ is the local Grashof number, $M^2 = \sigma B_0^2/(a\rho)$ is the magnetic parameter, and $P_r = \rho\upsilon C_p/k$ is the Prandtl Number.
The continuity equation (1) is satisfied identically, and the boundary conditions become

$$F(0) = 0,\ F'(0) = 1,\ \theta(0) = 1,\ F'(\infty) = 0 \quad and \quad \theta(\infty) = 0\ . \qquad (10)$$

In the above differential equation F represent the velocity coefficient of fluid and θ be the temperature coefficient of the fluid.

B: Principle of Optical Homotopy Asymptotic Method:-

The principles of OHAM as expanded by Marinca and Herisau [21] and other researchers.
Let us assume the following Non-linear differential equation and boundary conditions are

$$L\big(U(x)\big) + G(x) + N\big(U(x)\big) = 0, B(x) = 0, \quad (11)$$

where U(x) is an unknown function, G(x) is a known function, N is a nonlinear operator, L is a linear operator, and B is a boundary operator. With OHAM is constructed the set of equations:

$$(1-p)[L(\emptyset(x,p) + G(x)] = H(p)\big[L\big(\emptyset(x,p)\big) + G(x) + N\big(\emptyset(x,p)\big)\big], \qquad (12)$$

$$B\big(\emptyset(x,c)\big) = 0\ . \qquad (13)$$

Here the value is in between $[0 \le p \le 1]$, and it is embedding parameter, $H(p)$ is a nonzero ausiliary function fro $p \ne 0$ and $H(0) = 0$, and $\emptyset(x,p)$ is unknown function. For $p = 0$ and $p = 1$, its holds that

$$\emptyset(x,0) = U_0 \ and \ \emptyset(x,1) = U(x). \qquad (14)$$

The Value of p increases from 0 to 1, and solution of $\emptyset(x,p)$ varies from $U_0(x)$ to the solution $U(x)$, where $U_0(x)$ is obtained from equation (2) for $p = 0$.

$$L\big(U_0(x)\big) + G(x) = 0, \quad B(U_0) = 0. \qquad (15)$$

In this the function of $H(p)$ is chosen in the form:

$$H(p) = pc_1 + p^2c_2 + p^3c_3 + p^4c_4\ , \qquad (16)$$

where $c_1, c_2, c_3, c_4, \ldots..$ are constants which is to be determined.
Expanding $\emptyset(x,p,c_i)$ in Taylor's series expansion about p, one has been written as:

$$\emptyset(x,p,c_i) = U_0(x) + \textstyle\sum_{k=1}^{\infty} U_k(x,c_i)p^k, \quad i = 1,2,3 \qquad (17)$$

Now using equation (6) and (7) into equation (2), collecting the same powers of p, and equating each coefficient of p to zero, and then obtained the sets of linear differential equation with boundary conditions. Then solving the differential equation using boundary conditions, zeroth-, first-, and second-order solutions $U_0(x), U_1(x,c_1), U_2(x,c_1,c_2), \ldots....$ can be obtained.
The solution of the differential equation (1) is determined as

$$\overline{U} = U_0(x) + \textstyle\sum_{k=1}^{m} U_k(x,c_i). \qquad (18)$$

Substituting equation (7) in equation (1) results in the following residual

$$R(x,c_i) = L\big(\overline{U}(x,c_i)\big) + G(x) + N(\overline{U}(x,c_i)), \qquad (19)$$

if $R(x,c_i) = 0$, then $\overline{U}$ will be the exact solution. For nonlinear problems, generally this will not be the case.
For determining c_i $(i = 1,2,3,\ldots..,m)$, using the method of least squares

$$\frac{\partial P}{\partial c_1} = \frac{\partial P}{\partial c_2} = \frac{\partial P}{\partial c_3} = \cdots = \frac{\partial P}{\partial c_m} = 0, \qquad (20)$$

where P is the minimum error, with these conditions we can find the approximate solution of m[th] order. The system of non-linear differential equation (8) and (9) is a coupled system of Non-Linear ordinary differential equations and it is difficult to solve by the common methods of solution of the system of ordinary differential equation. We want to solve the Non-Linear ordinary differential equation by OHAM method, which basic principle is given above.

III. METHODS OF SOLUTION

In this section, the optimal homotopy asymptotic method (OHAM) is applied the solution of non linear differential equation. For this purpose we choose the set of bases functions to approximate the unknown functions $f(\eta)$ and $\theta(\eta)$ respectively as

$$F(\eta) = F_0(\eta) + pF_1(\eta) + p^2F_2(\eta) + \cdots \qquad (21)$$

$$\theta(\eta) = \theta_0(\eta) + p\theta_1(\eta) + p^2\theta_2(\eta) + \cdots \tag{22}$$

Putting the value of $F(\eta)$, $\theta(\eta)$ & $H(p)$ from equation (21), (22) and (16) in the differential equation (12) we get

$$(1-p)[F_0''' + pF_1''' + p^2F_2''' + \cdots] = (p\,c_1 + p^2\,c_2 + \cdots)[(F_0''' + pF_1''' + p^2F_2''' + \cdots) + (F_0 + pF_1 + p^2F_2 + \cdots)(F_0'' + pF_1'' + p^2F_2'' + \cdots) - (F_0' + pF_1' + p^2F_2' + \cdots)^2 + G_r(\theta_0 + p\theta_1 + p^2\theta_2 + \cdots) - M^2(F_0' + pF_1' + p^2F_2' + \cdots)], \tag{23}$$

$$(1-p)(\theta_0'' + p\theta_1'' + p^2\theta_2'' + \cdots) = (p\,c_1 + p^2\,c_2 + \cdots)[(\theta_0'' + p\theta_1'' + p^2\theta_2'' + \cdots) + P_r(F_0 + pF_1 + p^2F_2 + \cdots)(\theta_0' + p\theta_1' + p^2\theta_2' + \cdots)], \tag{24}$$

Equating the coefficient of p^0, p^1, p^2 ... from the above equation (23) & (24), we get

$$F_0''' = 0, \tag{25}$$

$$F_1''' = c_1(F_0F_0'' - F_0'^2 + G_r\,\theta_0 - M^2F_0') + (1 + c_1)F_0''', \tag{26}$$

$$F_2''' = c_1(F_1F_0'' + F_0F_1'' - 2F_0'F_1' + G_r\theta_1 - M^2F_1') + c_2(F_0F_0'' - F_0'^2 + G_r\theta_0 - M^2F_0') + (1 + c_1)F_1''' + c_2F_0''', \tag{27}$$

$$\theta_0'' = 0, \tag{28}$$

$$\theta_1'' = c_1P_r\,F_0\theta_0 + (1 + c_1)\theta_0'', \tag{29}$$

$$\theta_2'' = c_1(P_r\,F_1\theta_0' + P_r\,F_0\,\theta_1') + c_2P_r\,F_0\,\theta_0' + (1 + c_1)\theta_1'' + c_2\theta_0''. \tag{30}$$

The boundary conditions are

$$F(0) = 0,\ F'(0) = 1,\ F'(\infty) = 0,\ \theta(0) = 1,\ \theta(\infty) = 0, \tag{31}$$

Now integrating the zeros, first and Second order differential equation (25), (26), (27), (28), (29), (30) under the boundary condition (31), we get

$$\theta_0(\eta) = 1 - 0.1\,\eta, \tag{32}$$

$$F_0(\eta) = \eta - 0.05\,\eta^2, \tag{33}$$

$$\theta_1 = c_1P_r(0.00025\,\eta^5 - 0.0125\,\eta^4 + 0.1666667\,\eta^3) - (6.666667\,c_1P_r + 0.1)\eta + 1, \tag{34}$$

$$F_1 = c_1[-0.00008333\,\eta^5 - 0.00416667\,(G_r - M^2 - 1)\,\eta^4 + 0.1666667\,(G_r - M^2 - 1)\,\eta^3] + [c_1\{0.2083333 - 0.16666667\,(G_r - M^2 - 1)\} - 0.05]\eta^2 + \eta, \tag{35}$$

$$\theta_2 = -0.0000011161\,c_1^2\,P_r^2\,\eta^8 + c_1^2(0.000089286\,P_r^2 + 0.000000198413\,P_r)\,\eta^7 + c_1^2\{0.00001388889\,P_r(G_r - M^2 - 1) - 0.0025\,P_r^2\}\eta^6 - c_1^2\{0.00083333\,(G_r - M^2 - 1) - 0.025\}\eta^5 - [c_1^2\{-0.026042\,P_r^2 - 0.01388889\,P_r(G_r - M^2 - 1)\} - 0.00083333\,c_1P_r - 0.0000416667\,c_2P_r]\,\eta^4 + (0.111111\,c_1^2\,P_r^2 + 0.0333333\,c_1\,P_r + 0.0166667\,c_1\,P_r)\,\eta^3 + \chi_1\,\eta + 1, \tag{36}$$

$$F_2 = c_1^2(0.000000198413 + 0.000007441\,P_r\,G_r)\eta^8 + c_1^2(0.0000039683\,G_r - 0.00000198413\,M^2 + 0.00000793651 - 0.000059524\,P_rG_r)\eta^7 + c_1^2\{0.00013888889\,(M^2 + 73)(G_r - M^2 - 1) - 0.0013888889P_rG_r\}\eta^6 + \{c_1^2(0.0006111 - 0.002777778(3M^2 - 2)(G_r - M^2 - 1)) - 0.0001666667\,(3c_1 + 2c_2)\}\eta^5 - \{c_1^2(-0.01388889(M^2 + 1)(G_r - M^2 - 1.3125) + 0.04166667(G_r - M^2 - 1) + 0.2777778\,P_r\,G_r) + 0.004166667\,c_1(2G_r - 2M^2 - 3) + 0.004166667c_2(G_r - M^2 - 1)\}\eta^4 - 0.16666667\{c_1^2(G_r - M^2 - 1) + c_1(2G_r - 2M^2 - 3) + c_2(G_r - M^2 - 1)\}\eta^3 + \frac{\chi_2}{2}\,\eta^2 + \eta, \tag{37}$$

Where χ_1 & χ_2 is known constant, which is given bellow

$\chi_1 = -\{c_1^2(115.0777779\ P_r^2 - 6.7460319\ P_r + 6.9444445\ P_r\ G_r - 6.9444448\ P_r\ M^2) + 4.166666\ c_1\ P_r + 2.0833337\ c_2\ P_r + 0.1\}$, (38)

$\chi_2 = \{c_1^2(636.1110869\ M^2 + 19.444445\ G_r\ M^2 + 285.6349211\ P_r\ G_r - 630.5555541\ G_r + 641.74601) + c_1(6.666667\ M^2 - 6.666667\ G_r + 12.499999) + c_2(5\ G_r - 5M^2 - 3.333333) - 0.1\}$, (39)

Using the equation (32), (33), (34), (35), (36), (37) in equation (21) & (22), we get

$F(\eta) = \eta - 0.05\ \eta^2 + p\big[c_1\{-0.00008333\ \eta^5 - 0.00416667\ (G_r - M^2 - 1)\ \eta^4 + 0.1666667\ (G_r - M^2 - 1)\ \eta^3\} + \{c_1\big(0.2083333 - 0.16666667\ (G_r - M^2 - 1)\big) - 0.05\}\eta^2 + \eta\big] + p^2\Big[c_1^2(0.000000198413 + 0.000007441\ P_r\ G_r)\eta^8 + c_1^2(0.0000039683\ G_r - 0.00000198413\ M^2 + 0.00000793651 - 0.000059524\ P_rG_r)\eta^7 + c_1^2\{0.00013888889\ (M^2 + 73)(G_r - M^2 - 1) - 0.0013888889P_rG_r\}\eta^6 + \{c_1^2(0.0006111 - 0.002777778(3M^2 - 2)(G_r - M^2 - 1)) - 0.0001666667\ (3c_1 + 2c_2)\}\eta^5 - \{c_1^2(-0.1388889(M^2 + 1)(G_r - M^2 - 1.125) + 0.004166667(G_r - M^2 - 1) + 0.2777778\ P_r\ G_r) + 0.004166667\ c_1(2G_r - 2M^2 - 3) + 0.004166667c_2(G_r - M^2 - 1)\}\eta^4 - 0.16666667\{c_1^2(G_r - M^2 - 1) + c_1(2G_r - 2M^2 - 3) + c_2(G_r - M^2 - 1)\}\eta^3 + \frac{\chi_2}{2}\ \eta^2 + \eta\Big] + \cdots$ (40)

$\theta(\eta) = 1 - 0.1\ \eta + p[c_1P_r(0.00025\ \eta^5 - 0.0125\ \eta^4 + 0.1666667\ \eta^3) - (6.666667\ c_1P_r + 0.1)\eta + 1] + p^2\big[-0.0000011161\ c_1^2\ P_r^2\ \eta^8 + c_1^2(0.000089286\ P_r^2 + 0.000000198413\ P_r)\ \eta^7 + c_1^2\{0.00001388889\ P_r(G_r - M^2 - 1) - 0.0025\ P_r^2\}\eta^6 - c_1^2\{0.00083333\ (G_r - M^2 - 1) - 0.025\}\eta^5 - \big\{c_1^2\big(-0.026042\ P_r^2 - 0.01388889\ P_r(G_r - M^2 - 1)\big) - 0.00083333\ c_1P_r - 0.0000416667\ c_2P_r\big\}\ \eta^4 + (0.111111\ c_1^2\ P_r^2 + 0.0333333\ c_1\ P_r + 0.0166667\ c_1\ P_r)\ \eta^3 + \chi_1\ \eta + 1\big] + \cdots$ (41)

Now putting the value of $P_r = 0.7$, $G_r = 0.5$ and $M^2 = 0.1$ in equation (38 a), (39 a), (40) & (41)

$\chi_1 = -(54.0478333\ c_1^2 + 2.9166662\ \ c_1 + 1.45833359\ c_2 + 0.1)$, (42)

$\chi_2 = (432.8987884\ c_1^2 + 9.23333217\ c_1 - 0.883333\ c_2 - 0.1)$, (43)

$F(\eta) = \eta - 0.05\ \eta^2 + p[c_1\{-0.00008333\ \eta^5 + 0.002125002\ \eta^4 - 0.085\ \ \eta^3\} + (0.2933333\ c_1 - 0.05)\eta^2 + \eta] + p^2[0.000002802763\ c_1^2\ \eta^8 - 0.0000109325813\ c_1^2\ \eta^7 + 0.005657652823\ c_1^2\ \eta^6 - (0.002123066885\ c_1^2 + 0.0005\ c_1 + 0.000333334\ c_2)\eta^5 - (0.0184173625\ c_1^2 - 0.00841666734\ c_1 + 0.002125\ c_2)\eta^4 + (0.085\ c_1^2 + 0.336666667\ c_1 + 0.085\ c_2)\eta^3 + (216.4493942\ c_1^2 + 4.616666085\ c_1 - 0.4416665\ c_2 - 0.05)\ \eta^2 + \eta] + \cdots$ (44)

$\theta(\eta) = 1 - 0.1\ \eta + p[0.000175\ c_1\ \eta^5 - 0.00875c_1\ \eta^4 + 0.11666669\ c_1\ \eta^3 - (4.6666669\ c_1 + 0.1)\eta + 1] + p^2[-0.000000546889\ c_1^2\ P_r^2\ \eta^8 + 0.00004388903c_1^2\ \eta^7 - 0.000029958334\ c_1^2\ \eta^6 + 0.025425\ c_1^2\ \eta^5 + (0.00780225\ c_1^2 + 0.000583331\ c_1 + 0.0000291667\ c_2)\ \eta^4 + (0.0544444\ c_1^2 + 0.0233333\ c_1\ \ + 0.01166669\ c_2)\ \eta^3 - (54.0478333\ c_1^2 + 2.9166662\ \ c_1 + 1.45833359\ c_2 + 0.1)\ \eta + 1] + \cdots$ (45)

Using the method of least square in equation (44) & (45), we get the value of

For F
$c_1 = 0.00065489$ & $c_2 = -1.2309855$
For θ
$c_1 = 0.0000019881$ & $c_2 = 0.000000003883$

Putting these values in equation (40) & (41), we get

$$F(\eta) = \eta - 0.05\,\eta^2 + p[-0.00000065489\,\eta^5 - 0.000002728711\,(G_r - M^2 - 1)\,\eta^4 + 0.000109148355\,(G_r - M^2 - 1)\,\eta^3 + \{0.0001364354 - 0.00011091483355\,(G_r - M^2 - 1) - 0.05\}\eta^2 + \eta] + p^2[(0.00000000000008509555 + 0.000000000003191303\,P_r\,G_r)\eta^8 + (0.00000000000170193\,G_r - 0.0000000000008509555\,M^2 + 0.00000000000340382 - 0.00000000002552871\,P_r G_r)\eta^7 + \{0.0000000000595668\,(M^2 + 73)(G_r - M^2 - 1) - 0.00000000059566792 P_r G_r\}\eta^6 + \{0.000410001399 - 0.000000001191336(3M^2 - 2)(G_r - M^2 - 1)\}\eta^5 - \{-0.0000000595668\,(M^2 + 1)(G_r - M^2 - 1.125) + 0.0000001191336\,P_r\,G_r - 0.005123647456\,G_r + 0.005123647455\,M^2 + 0.005120918747\}\eta^4 - 0.16666667(-1.229675291\,G_r + 1.229675291\,M^2 + 1.229020401)\,\eta^3 + (0.00000833935131\,G_r\,M^2 + 0.0001225033655\,P_r\,G_r - 3.079781933\,G_r + 3.079510309\,M^2 + 2.005872973)\,\eta^2 + \eta] + \cdots \tag{46}$$

$$\theta(\eta) = 1 - 0.1\,\eta + p[P_r(0.000000000497025\,\eta^5 - 0.00000002485125\eta^4 + 0.0000003313501\,\eta^3) - (0.000013254001\,P_r + 0.1)\eta + 1] + p^2[-0.00000000000000000004411432\ \ P_r^2\,\eta^8 + (0.000000000000000035290663\,P_r^2 + 0.00000000000000000078423564\,P_r)\,\eta^7 + \{0.000000000000000005489416\,P_r(G_r - M^2 - 1) - 0.0000000000000009881354\,P_r^2\}\eta^6 - \{0.0000000000000032937715\,(G_r - M^2 - 1) - 0.0000000000000009881354\}\eta^5 + \{0.0000000000000102932089\,P_r^2 + 0.0000000000000054894156\,P_r(G_r - M^2 - 1) + 0.0000000000000165085568\,P_r\}\,\eta^4 + (0.0000000000000439170851\ \ P_r^2 + 0.0000000000648485473\,P_r)\,\eta^3 - \{(0.00000000045480395\,P_r^2 - 0.00000000002666397\,P_r + 0.00000000002744821\,P_r\,G_r - 0.00000000002744821\,P_r\,M^2) + 0.0000082837487\ \ P_r + 0.0000000080895848\ \ P_r + 0.1\}\,\eta + 1] + \cdots \tag{47}$$

$$F'(\eta) = \eta - 0.05\,\eta^2 + p[-0.00000065489\,\eta^5 - 0.000002728711\,(G_r - M^2 - 1)\,\eta^4 + 0.000109148355\,(G_r - M^2 - 1)\,\eta^3 + \{0.0001364354 - 0.0001091483355\,(G_r - M^2 - 1) - 0.05\}\eta^2 + \eta] + p^2[(0.00000000000008509555 + 0.000000000003191303\,P_r\,G_r)\eta^8 + (0.00000000000170193\,G_r - 0.0000000000008509555\,M^2 + 0.00000000000340382 - 0.00000000002552871\,P_r G_r)\eta^7 + \{0.0000000000595668\,(M^2 + 73)(G_r - M^2 - 1) - 0.00000000059566792 P_r G_r\}\eta^6 + \{0.000410001399 - 0.000000001191336(3M^2 - 2)(G_r - M^2 - 1)\}\eta^5 - \{-0.0000000595668\,(M^2 + 1)(G_r - M^2 - 1.125) + 0.0000001191336\,P_r\,G_r - 0.005123647456\,G_r + 0.005123647455\,M^2 + 0.005120918747\}\eta^4 - 0.16666667(-1.229675291\,G_r + 1.229675291\,M^2 + 1.229020401)\,\eta^3 + (0.00000833935131\,G_r\,M^2 + 0.0001225033655\,P_r\,G_r - 3.079781933\,G_r + 3.079510309\,M^2 + 2.005872973)\,\eta^2 + \eta] + \cdots \tag{48}$$

$$\theta'(\eta) = 1 - 0.1\,\eta + p[P_r(0.000000000497025\,\eta^5 - 0.00000002485125\eta^4 + 0.0000003313501\,\eta^3) - (0.000013254001\,P_r + 0.1)\eta + 1] + p^2[-0.000000000000000004411432\;P_r^2\,\eta^8 + (0.0000000000000000035290663\,P_r^2 + 0.000000000000000000078423564\,P_r)\,\eta^7 + \{0.0000000000000000005489416\,P_r(G_r - M^2 - 1) - 0.0000000000000000009881354\,P_r^2\}\eta^6 - \{0.000000000000000032937715\,(G_r - M^2 - 1) - 0.0000000000000009881354\}\eta^5 + \{0.000000000000102932089\,P_r^2 + 0.0000000000000054894156\,P_r(G_r - M^2 - 1) + 0.00000000000000165085568\,P_r\}\,\eta^4 + (0.000000000000439170851\;P_r^2 + 0.000000000000648485473\,P_r)\,\eta^3 + \chi_1\,\eta + 1] + \quad (49)$$

$$F''^{(\eta)} = \eta - 0.05\,\eta^2 + p[-0.00000065489\,\eta^5 - 0.000002728711\,(G_r - M^2 - 1)\,\eta^4 + 0.000109148355\,(G_r - M^2 - 1)\,\eta^3 + \{0.0001364354 - 0.0001091483355\,(G_r - M^2 - 1) - 0.05\}\eta^2 + \eta] + p^2[(0.0000000000000008509555 + 0.000000000003191303\,P_r\,G_r)\eta^8 + (0.00000000000170193\,G_r - 0.0000000000000008509555\,M^2 + 0.00000000000340382 - 0.00000000002552871\,P_r G_r)\eta^7 + \{0.0000000000595668\,(M^2 + 73)(G_r - M^2 - 1) - 0.00000000059566792 P_r G_r\}\eta^6 + \{0.000410001399 - 0.000000001191336(3M^2 - 2)(G_r - M^2 - 1)\}\eta^5 - \{-0.0000000595668\,(M^2 + 1)(G_r - M^2 - 1.125) + 0.0000001191336\,P_r\,G_r - 0.005123647456\,G_r + 0.005123647455\,M^2 + 0.005120918747\}\eta^4 - 0.16666667(-1.229675291\,G_r + 1.229675291\,M^2 + 1.229020401)\,\eta^3 + (0.00000833935131\,G_r\,M^2 + 0.0001225033655\,P_r\,G_r - 3.079781933\,G_r + 3.079510309\,M^2 + 2.005872973)\,\eta^2 + \eta] + \quad (50)$$

$$\theta''^{(\eta)} = 1 - 0.1\,\eta + p[P_r(0.000000000497025\,\eta^5 - 0.00000002485125\eta^4 + 0.0000003313501\,\eta^3) - (0.000013254001\,P_r + 0.1)\eta + 1] + p^2[-0.000000000000000004411432\;P_r^2\,\eta^8 + (0.0000000000000000035290663\,P_r^2 + 0.000000000000000000078423564\,P_r)\,\eta^7 + \{0.0000000000000000005489416\,P_r(G_r - M^2 - 1) - 0.0000000000000000009881354\,P_r^2\}\eta^6 - \{0.000000000000000032937715\,(G_r - M^2 - 1) - 0.0000000000000009881354\}\eta^5 + \{0.000000000000102932089\,P_r^2 + 0.0000000000000054894156\,P_r(G_r - M^2 - 1) + 0.00000000000000165085568\,P_r\}\,\eta^4 + (0.000000000000439170851\;P_r^2 + 0.000000000000648485473\,P_r)\,\eta^3 + \chi_1\,\eta + 1] + \cdots \quad (51)$$

IV. RESULTS AND DISCUSSION

The objective of the present study is to find out the solution of MHD flow of fluid and heat transfer problem by optical homotopy asymptotic method. In this paper the graph of velocity against distance has been plotted using constant parameter p, k and M. Fig.1. is the graph of width of the channel against velocity of fluid and show that with the increase of the embedding parameter p, axial velocity of flow increase sharply, whereas increase of parameter k, do not affect the axial velocity of fluid, which has been shown in Fig. 2.

Fig.3. is the graph of width of the channel against temperature change of the fluid and show the variation of embedding parameter p. With the increase of the embedding parameter p the temperature of the fluid decreases slowly, whereas increase of parameter k, not affected the axial velocity of fluid, which is shown in Fig. 4. Fig.5. is the graph of width of the channel against the velocity of fluid and

temperature of the fluid. This graph reveal the variation of embedding parameter p and it is observed that with the increase of the embedding parameter p the velocity of fluid and heat transfer of the fluid decreases slowly.

The graph of width of the channel against radial velocity of fluid has been shown in Fig. 6. It is observed that with the increase of the embedding parameter p the radial velocity of fluid increases.

Fig.7. is the graph of width of the channel against $f(\eta)$, $f'(\eta)$ & $f''(\eta)$ at embedding parameter p = 0.3 and p = 0.8. This figure shows that with the increase of p then the value of $f(\eta)$, $f'(\eta)$ & $f''(\eta)$ increases. It is observed that the compared values of $f'(\eta)$ & $f''(\eta)$ in regard to $f(\eta)$ decreases. It has also been observed that order of derivative of heat transfer of fluid $\theta(\eta)$, $\theta'(\eta)$ & $\theta''(\eta)$ decreases, which has been shown in the figure 8, at p = 0.8. From Fig 9; it is observed that the increase of Magnetic parameter M, axial velocity of fluid sharply increases where as radial velocity of fluid slowly increases.

Fig.10, is the graph of width η of the channel against heat transfer of fluid; it is shown that increase of magnetic parameter M, there is no change of heat transfer of fluid.

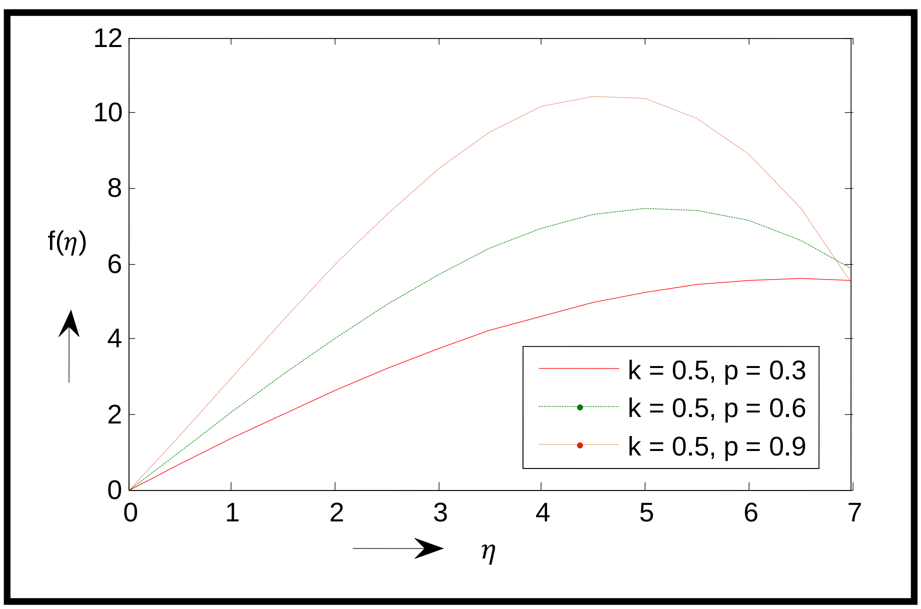

Fig. 1. Graph between dimensionless variable η and velocity of fluid $f(\eta)$; this graph shows that the variation of embedding parameter p at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

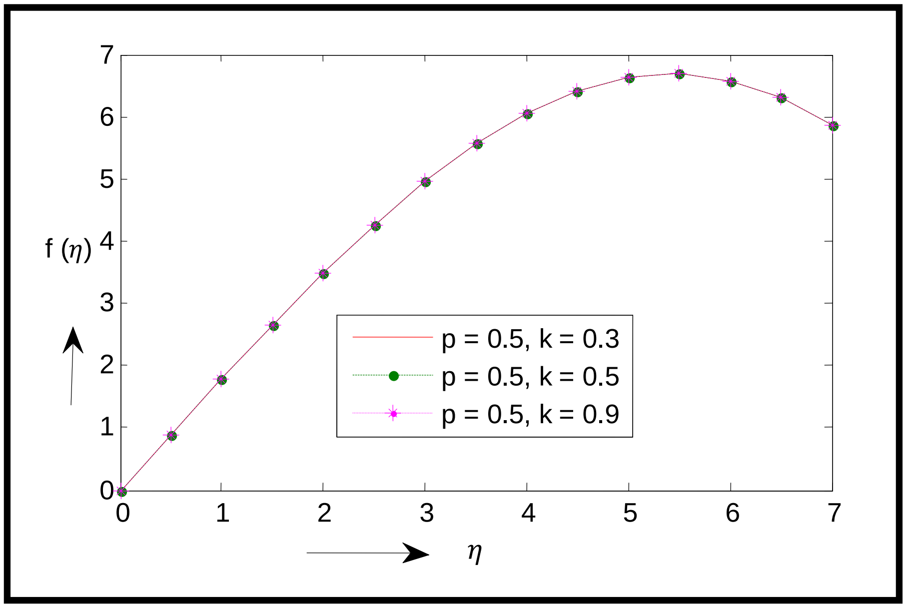

Fig. 2. Graph between dimensionless variable η and velocity of fluid $f(\eta)$; this graph shows that the variation parameter k at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

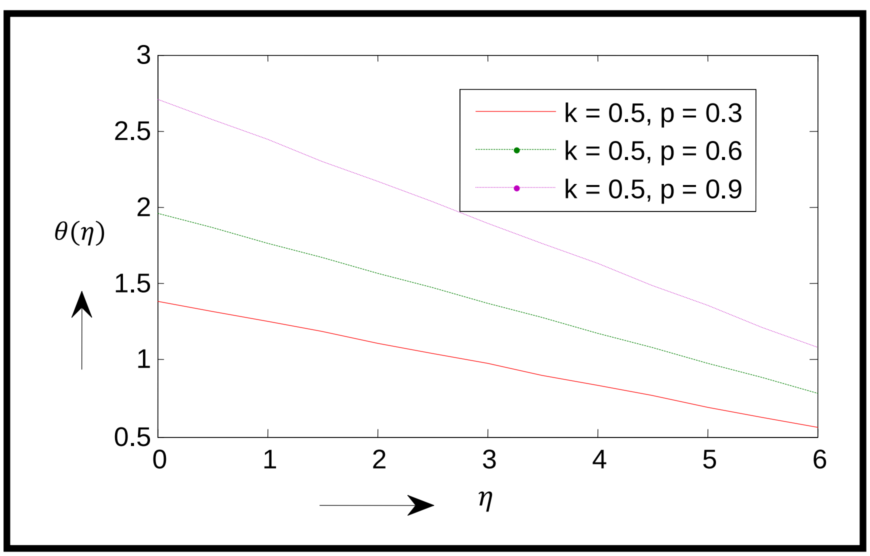

Fig. 3. Graph between dimensionless variable η and velocity of fluid $\theta(\eta)$; this graph shows that the variation embedding parameter p at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

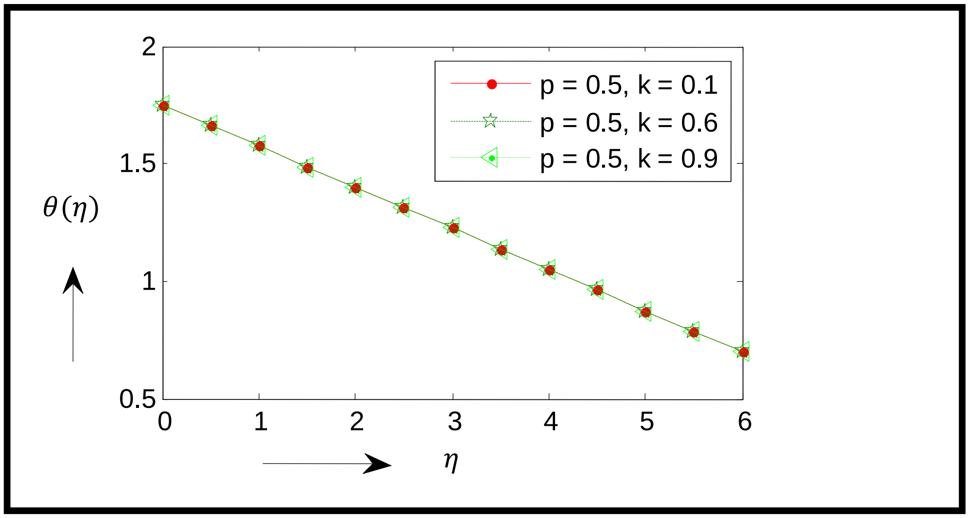

Fig. 4. Graph between dimensionless variable η and velocity of fluid $\theta(\eta)$; this graph shows that the variation parameter k at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

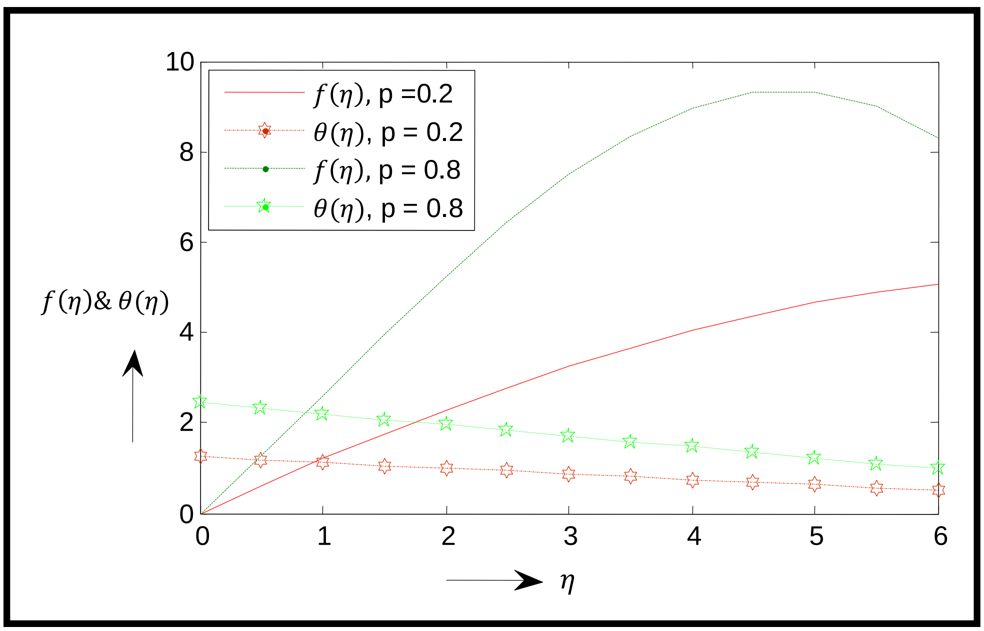

Fig. 5. Graph between dimensionless variable η and velocity of fluid $f(\eta), \theta(\eta)$; this graph shows that the variation of embedding parameter p at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

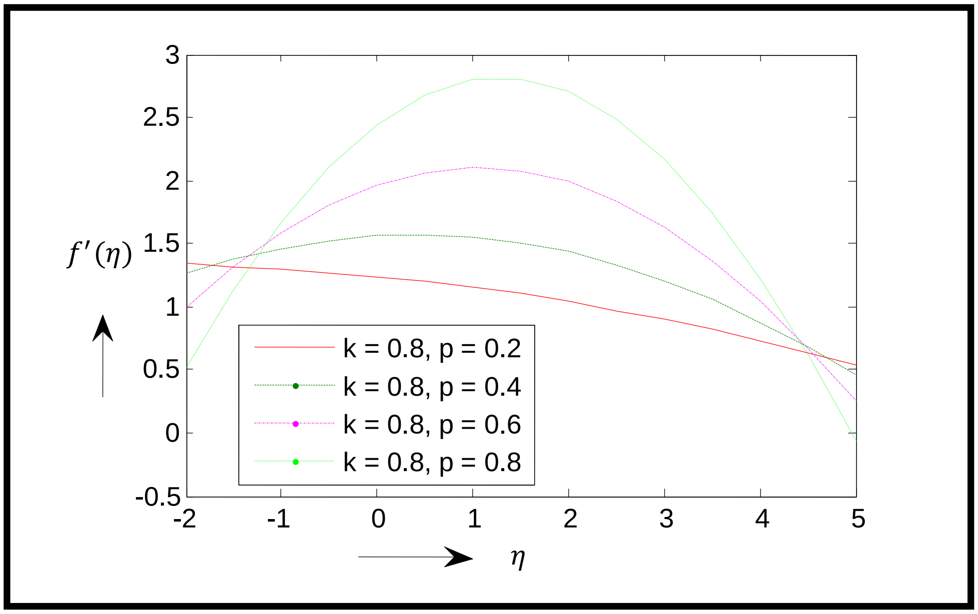

Fig. 6. Graph between dimensionless variable η and radial velocity of fluid $f'(\eta)$; this graph shows that the variation of embedding parameter p at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

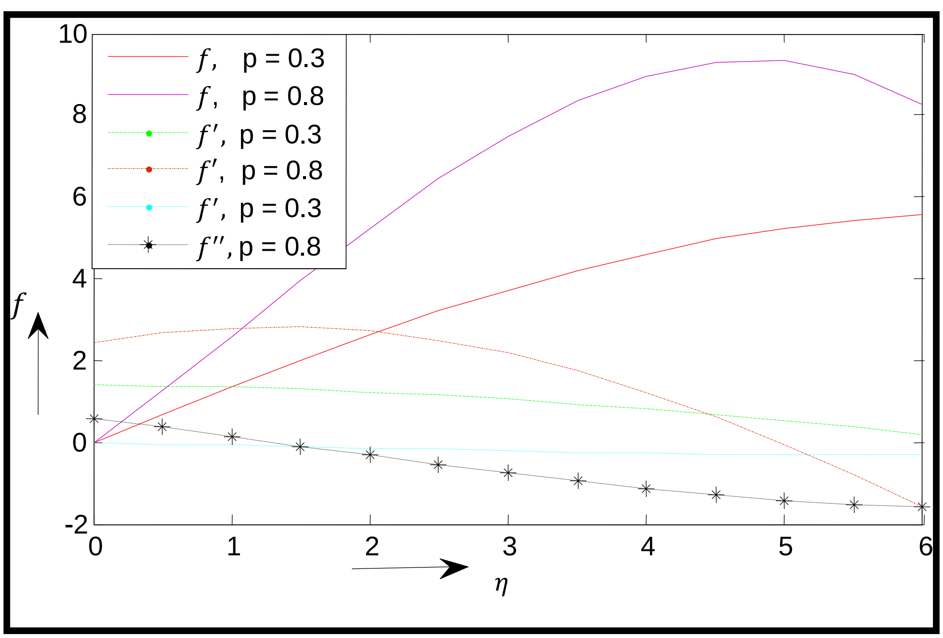

Fig. 7. Graph between dimensionless variable η and $f(\eta)$, $f'(\eta)$ & $f''(\eta)$; this graph shows that the variation of embedding parameter p at constant value of Pr = 0.7, Gr = 0.5 and M = 0.1.

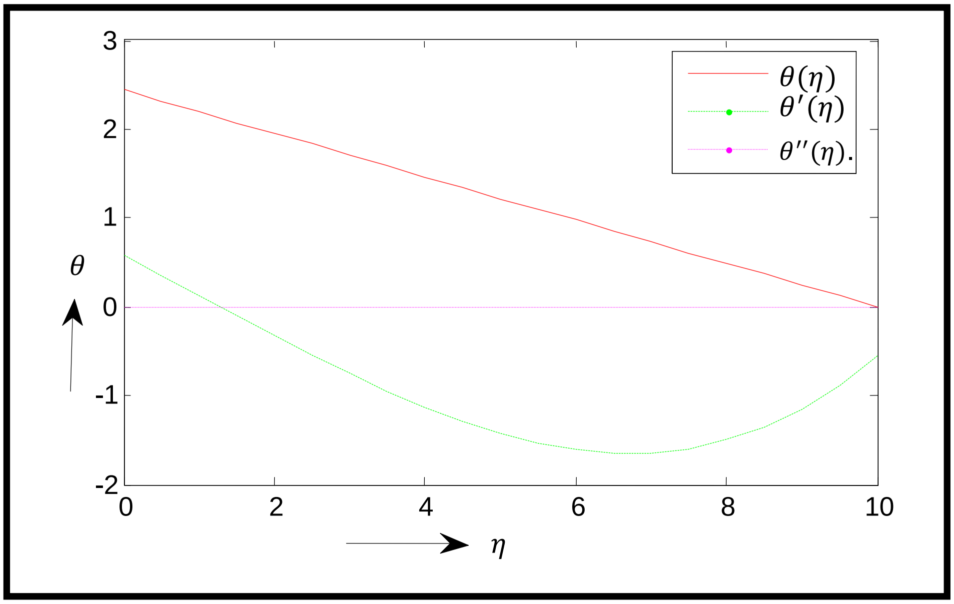

Fig. 8. Graph between dimensionless variable η and $\theta(\eta), \theta'(\eta) \& \theta''(\eta)$; this graph shows that the variation of derivative of θ at constant embedding parameter p = 0.8 and other constant parameter k = 0.5, Pr = 0.7, Gr = 0.5 and M = 0.1.

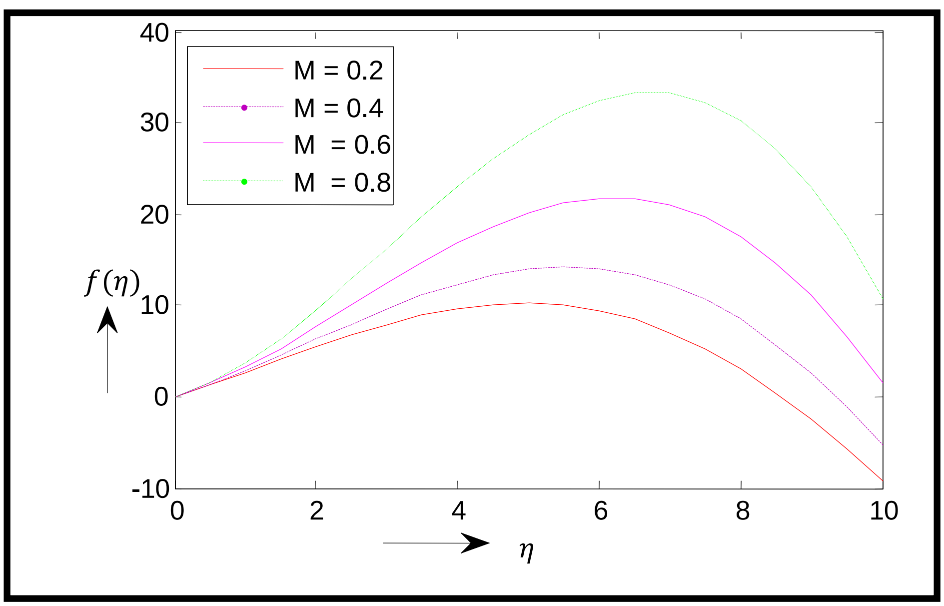

Fig. 9. Graph between dimensionless variable η and axial velocity of fluid f(η); this graph shows that the variation magnetic parameter M at constant embedding parameter p = 0.8 and other parameter k = 0.5, Pr = 0.7, Gr = 0.5 and M = 0.1.

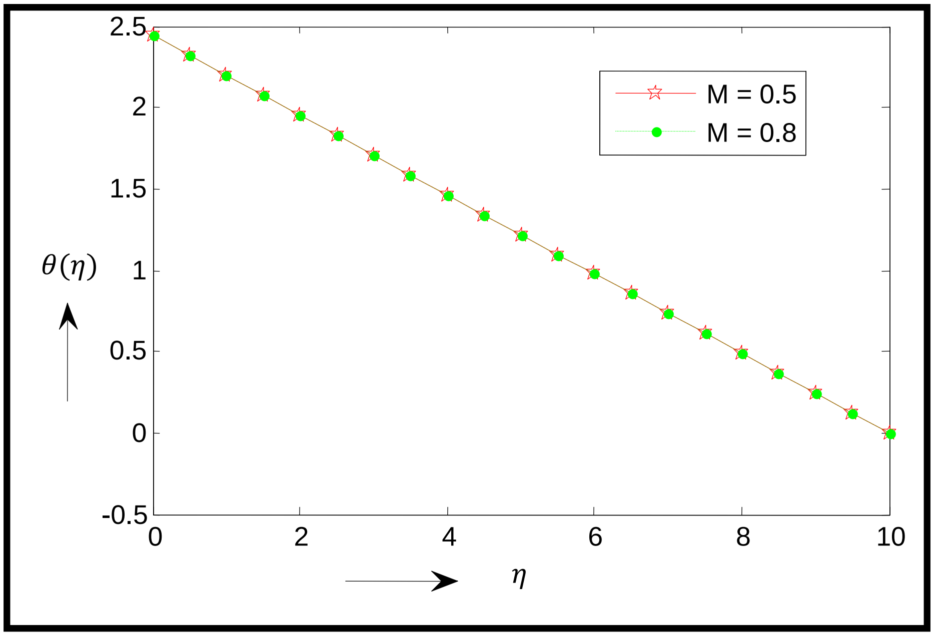

Fig. 10. Graph between dimensionless variable η and heat transfer of fluid $\theta(\eta)$; this graph shows that the variation magnetic parameter M at constant embedding parameter p = 0.8 and other parameter k = 0.5, Pr = 0.7, Gr = 0.5.

V. Conclusion

In this article, the solution of MHD flow of fluid and Heat transfer study in fluid by Optima homotopy asymptotic method has been investigated. The main objective is to investigation the effect of k, embedding parameter p and magnetic parameter M on the velocity of fluid and heat transfer studied with bounding wall. With the increase of embedding parameter p, the axial & radial velocity of fluid increase sharply, whereas the reciprocal affect on the heat flow of fluid is observed. On the other hand with the increase of Magnetic parameter M, axial and radial velocity of fluid increase sharply and opposite effect of heat flow of fluid this has been shown in figures. Such types of flows have applications in engineering and biological problems such as accelerators electrostatics filtration, preparation polymer, petroleum industry and purification of crude oil and plasma studies.

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