A proposed new algorithm in FTP

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ABSTRACT - This paper proposes a new method to find the optimal solution to transportation problems in fuzzy environments, with the help of triangular fuzzy numbers using the techniques of zero termination method, measure of fuzzy number and new arithmetic operations.

Keywords: Triangular fuzzy number, Fuzzy triangular membership function, Fuzzy Transportation Problem (FTP), α -cuts: Zero termination method.

1. Introduction

The transportation problem is a special linear programming problem which arises in many practical applications. Many problems which have nothing to do with transportation have this structure. Suppose that "m" origins a certain product supply to "n" destinations. Let a_i be the amount of the product available at origin i, and b_j be the quantity of the product required at destination j, Further, we assume that the cost of shipping a unit quantity of the product from origin i to destination j is c_{ij}, Let x_{ij} represent the quantity shipped from origin i to destination j. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Shiang-Tai liu and Chiang Kao [1], Chanas et al. [2], Chanas and Kuchta [3] proposed a method for solving fuzzy transportations problems to obtain crisp solutions. Nagoor gani and Abdul Razak [4] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem, in which fuzzy supplies and demands are triangular fuzzy numbers.

In most of the cases in our life, the data obtained for decision making are only imprecise. In 1965, Zadeh [5] introduced the concept of fuzzy set theory to meet these problems. In 1978, Dubols and Prade [6] defined fuzzy numbers as a fuzzy subset of the real line. In 1988, J.J. Buckley [7] has introduced the concept of fuzzy triangular numbers to linear programming. In a fuzzy transportation problem all the parameters are fuzzy numbers. Fuzzy numbers may be normal (or) abnormal, triangular numbers. Since, some fuzzy numbers are not directly comparable, comparing two (or) multi fuzzy numbers and ranking such numbers becomes important. Here we discuss the setting the rank of fuzzy numbers. In this paper we propose the method of zero termination, and new arithmetic operation to find the optimal solution for the total FTP in the nature of the fuzzy triangular membership function.

This paper is organized as: In section 2, some basic definitions on fuzzy set theory are listed. In section 3, α -cut, new arithmetic operation, necessary existence conditions are discussed. In section 4, Ranking fuzzy number, measures of fuzzy numbers are discussed. In section 5, Zero termination method, Algorithms to find out the optimal solution for the total fuzzy transportation on minimum cost is discussed. In section 6, a new numerical example is worked out.

2. Preliminaries

2.1. Definition: [5] A fuzzy set A is defined by $A=\{(x,\mu_A(x)/x\in A,\mu_A(x)\in[0,1]\}$.In this pair $(x,\mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belongs to the interval [0, 1], called membership function.

2.2. Definition: [2] The support of fuzzy set A is the set of all points x in X, such that $\mu_A(x)>0$, i.e. $A=\{x/\mu_A(x)>0\}$

2.3. Definition: [5] The α -cut (or) α -level set of fuzzy subset A is a set consisting of those elements of the universe X whose membership values exceed the threshold level α i.e, $A_\alpha=\{x/\mu_A(x)\geq\alpha\}$

2.4. Definition: [4] A triangular fuzzy number 'A' represent with the three points as follow $A=[\underline{a},a,\overline{a}]$.This representation is interpreted as membership function is

$$\mu_A(x)=\begin{cases}0 & for\ \ x<\underline{a}\\ \dfrac{x-\underline{a}}{a-\underline{a}} & for\ \underline{a}\leq x\leq a\\ \dfrac{\overline{a}-x}{\overline{a}-a} & for a\leq x\leq\overline{a}\\ 0 & for\ \ x>\overline{a}\end{cases}$$

2.5. Definition: [4] Let $A=[\underline{a},a,\overline{a}]$ and $B=[\underline{b},b,\overline{b}]$ are two triangular fuzzy numbers then the arithmetic operation is

Addition: $A+B=\left\{ \underline{a}+\underline{b}\ ,\ a+b,\ \overline{a}+\overline{b} \right\}$

Subtraction: $A-B=\left\{ \underline{a}-\overline{b}\ ,\ a-b,\ \overline{a}-\underline{b} \right\}$

Multiplication:

(i) If 'A' and 'B' are positive, then

$$A\bullet B=\left\{ a\underline{b}+b\underline{a}\ ,ab,a\overline{b}+b\overline{a} \right\}$$

(ii) If 'A' is negative and 'B' is positive, then $A=[\underline{a},-a,\overline{a}]$, $B=[\underline{b},b,\overline{b}]$

and $A\bullet B=\left\{ b\overline{a}-a\underline{b},\ ab, b\underline{a}-a\overline{b} \right\}$

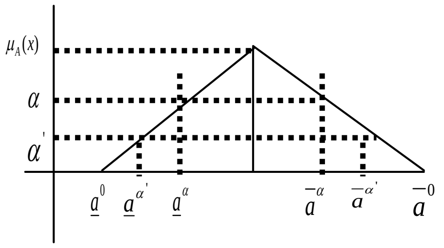
(iii) If 'A' & B' are negative, then $A=[\underline{a},-a,\overline{a}]$,

$B=[\overline{b},-b,\underline{b}]$ and

$$A\bullet B=\left\{ -a\overline{b}-b\overline{a}\ ,ab,-a\underline{b}-b\underline{a} \right\}$$

3. α -cut operation on fuzzy numbers

Generally a fuzzy interval is represented by two points $\underline{a},\overline{a}$ be the peak points of 'a' as $[\underline{a},a,\overline{a}]$. The α -cut operation can also be applied to a fuzzy number. If we denote the α -cut interval for the fuzzy number as A_α , the obtained interval A_α is defined as $A_\alpha=[\underline{a}^\alpha,\overline{a}^\alpha]$.



It is observed that $\alpha'<\alpha \Rightarrow a_{\alpha'}<a_\alpha$:

Indeed, $\underline{a}^{\alpha'}\le \underline{a}^{\alpha}$, $\overline{a}^{\alpha'}\le \overline{a}^{\alpha}$.

3.1 Definition: Operation on fuzzy numbers can be generalized from that of crisp interval as follows: $\forall\ a,b,\underline{a},\underline{b},\overline{a},\overline{b}\in R$

Let $A=[\underline{a},a,\overline{a}]$ and $B=[\underline{b},b,\overline{b}]$

$$A+B=\left\{ \underline{a}+\underline{b}\ ,\ a+b,\ \overline{a}+\overline{b} \right\}$$

$$A-B=\left\{ \underline{a}-\overline{b}\ ,\ a-b,\ \overline{a}-\underline{b} \right\}$$

$$A_\alpha=[\underline{a}^\alpha,\overline{a}^\alpha]=[(a-\underline{a})\alpha+\underline{a}\ ,\ (a-\overline{a})\alpha+\overline{a}]$$

$$B_\alpha=[\underline{b}^\alpha,\overline{b}^\alpha]=[(b-\underline{b})\alpha+\underline{b}\ ,\ (b-\overline{b})\alpha+\overline{b}],$$

product of α -cut is

$$R_\alpha=A_\alpha\times B_\alpha=[\underline{a}^{\alpha},\overline{a}^{\alpha}]\times[\underline{b}^{\alpha},\overline{b}^{\alpha}]$$

$$=[\underline{R}^\alpha,\overline{R}^\alpha]$$

Where

$$\underline{R}^\alpha=\min(\underline{a}^\alpha\underline{b}^\alpha,\underline{a}^\alpha\overline{b}^\alpha,\overline{a}^\alpha\underline{b}^\alpha,\overline{a}^\alpha\overline{b}^\alpha)\quad and$$

$$\overline{R}^\alpha=\max(\underline{a}^\alpha\underline{b}^\alpha,\underline{a}^\alpha\overline{b}^\alpha,\overline{a}^\alpha\underline{b}^\alpha,\overline{a}^\alpha\overline{b}^\alpha)$$

3.2 Definition: New operations of Addition and Subtraction on Triangular Fuzzy numbers:

If $A=[\underline{a},a,\overline{a}]$ & $B=[\underline{b},b,\overline{b}]$ are fuzzy numbers,

(i) Addition: $A+B=[\underline{a}+\overline{b}\ ,a+b,\ \overline{a}+\underline{b}]$

if $MP(A)\ge MP(B)$, where MP (A) denotes the midpoint of the triangular fuzzy number, $MP(A)=\frac{\underline{a}+\overline{a}}{2}$.

(ii) Subtraction: $A-B=[\underline{a}-\underline{b}\ ,a-b,\ \overline{a}-\overline{b}]$

if $DP(A)\ge DP(B)$, where DP (A) denotes the difference point of a triangular fuzzy number, $DP(A)=\frac{\overline{a}-\underline{a}}{2}$.

The following proposition establishes the validity of the above definitions under the conditions mentioned.

3.3 Proposition: **(1)** The new addition 3.2(i) is valid if the following condition is satisfied $MP\ (A)\ge MP\ (B)$.

(2) The new subtraction 3.2(ii) is valid if the following condition is satisfied $DP(A)\ge DP(B)$.

Proof: 1) If c = a + b is true, then

$$\begin{aligned}
\underline{c}\le\overline{c}\quad & iff\quad \underline{a}+\overline{b}\le\overline{a}+\underline{b}\\
& iff\quad \overline{b}-\underline{b}\le\overline{a}-\underline{a}\\
& iff\quad (MP(B)+DP(B)-(DP(B)-MP(B))\le\\
& \qquad (MP(A)+DP(A))-(DP(A)-MP(A))\\
& iff\quad 2MP(A)\le 2MP(A)\\
& iff\quad MP(A)\ge MP(B)
\end{aligned}$$

(2) if c = a-b is true, then $\underline{c}\le\overline{c}$,

$$\underline{c} \leq \overline{c} \text{ iff } \underline{a} - \underline{b} \leq \overline{a} - \overline{b}$$
$$\text{iff } \overline{b} - \underline{b} \leq \overline{a} - \underline{a}$$
$$\text{iff } (MP(B) + DP(B) - (MP(B) - DP(B)) \leq (MP(A) + DP(A)) - (MP(A) - DP(A))$$
$$\text{iff } 2DP(A) \leq 2DP(A)$$
$$\text{iff } DP(A) \geq DP(B)$$

4. Measure of a fuzzy number

Basirzadeh [13], A function $M : F(X) \rightarrow R^{+}$ where F(X) denotes the set of all fuzzy number on X. For each fuzzy number A, this function assigns a non-negative real number M (A) that expresses the measure of "A". The measure of a fuzzy number is obtained by the average of two side areas; left side and right side are from membership function to an axis the following requirements are essential,

4.1 Definition: [13] If $A = [\underline{a}, a, \overline{a}]$ is a Triangular Fuzzy Number, then $M^{TRI}(A) = \frac{1}{4}(2a + \underline{a} + \overline{a})$. It satisfies

$$A \leq B \text{ iff } M(A) \leq M(B)$$

5. Zero Termination Method.

The Algorithm of the proposed method of Zero termination is as follows:

Step 1: Construct the transportation table

Step 2: Select the very smallest unit transportation cost value for each row and subtract each entries of the transportation table from the corresponding row minimum after that using the similar way subtract each column entries of the transportation table from the corresponding column minimum

Step 3: In the reduced cost matrix, there will be at least one zero in each row and column, then find the termination value of all the Zero cells in the reduced cost matrix by the following simplification:

The termination cost T = Sum of the costs of all the cells adjacent to zero cell is divided by the number of non- zero cells added

Step 4: Choose the cell having maximum termination cost (T), if it has one maximum value, then first maximum possible allocation (demand) is made to the cell. If it has one (or) more equal values then select the cell having minimum cost of transportation and allocation is made.

Step 5: After the above step, the exhausted demands (column) (or) supplies (row) to be trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat step (2)

Step 6: Repeat step (3) to step (5) until the optimal solution is obtained.

5.1 New approach for solving fuzzy transportation problem

A new method for solving a fuzzy transportation problem with the transportation costs are the triangular fuzzy numbers is described as follows;

Step 1: Calculate the measure of Triangular fuzzy number which represents the transportation costs c_{ij} .

Step 2: Replace c_{ij} ,by M (c_{ij}).

Step 3: Now the new crisp transportation problem, is solved by using the zero termination method. Using new arithmetic operations, the optimal solution of Fuzzy transportation problem is obtained.

Numerical Examples

Consider the following fuzzy transportation problem of minimal cost representation Table 1; the optimal cost is obtained by the measure of fuzzy numbers, zero termination method. The problem is balanced fuzzy transportation problem (total fuzzy capacity value is equal to the total fuzzy demand value) and then supply and demand costs are symmetric fuzzy triangular number (FTN) by Table. 1

Table 1. the basic fuzzy transportation problem is

					Capcity
	(-2 0 2)	(0 1 2)	(-2 0 2)	(-1 0 1)	(0 1 2)
	(4 8 12)	(4 7 10)	(2 4 6)	(1 3 5)	(1 3 5)
	(2 4 6)	(4 6 8)	(4 6 8)	(4 7 10)	(2 7 12)
Demand	(1 3 5)	(0 2 4)	(1 3 5)	(1 3 5)	

By using the new method, it can be obtain the minimal transportation cost. (Table 2)

Table 2. the minimal transportation cost

					Capcity
	(-2 0 2) [0 1 2]	(0 1 2)	(-2 0 2)	(-1 0 1)	(0 1 2)
	(4 8 12)	(4 7 10)	(2 4 6)	(1 3 5) [1 3 5]	(1 3 5)
	(2 4 6) [1 2 3]	(4 6 8) [0 2 4]	(4 6 8) [1 3 5]	(4 7 10) [1 3 5]	(2 7 12)
Demand	(1 3 5)	(0 2 4)	(1 3 5)	(1 3 5)	

Since, the number of allocated cells is less than or equal to m+n-1 and also independent there exist a non-degenerate fuzzy basic solution, therefore the, minimum fuzzy transportation cost is

$\sum_{j,i} Z_{ij} = [\underline{z_{ij}}, z_{ij}, \overline{z_{ij}}]$ = [-2 0 2] [0 1 2] + [1 3 5] [1 3 5]

+ [2 4 6] [1 2 3] + [4 6 8] [0 2 4] + [4 6 8] [1 3 5]

+ [4 710][1 3 5]

= [11 68 169]

The minimum fuzzy transportation cost is 55 [using 4]

Computation of membership function:

Computation of membership functions of the fuzzy optimal solution of the fuzzy transportation problem. It is to find fuzzy membership functions of c_{ij} and μ_{ij} for each cell (i, j). The membership function of fuzzy transportation cost for the occupied cells are fuzzy allocation and their α - level sets of fuzzy transportation cost and fuzzy allocation as follows: $\mu_{c11}(x) = \begin{cases} \frac{x+2}{2} & -2 \le x \le 0 \\ \frac{x-2}{2} & 0 \le x \le 2 \end{cases}$,

$$\mu_{x11}(x) = \begin{cases} \frac{x-0}{1} & 0 \le x \le 1 \\ \frac{2-x}{1} & 1 \le x \le 2 \end{cases}$$

$$c_{11}^{\alpha} = (2\alpha - 2, 2 - 2\alpha)\ , \quad x_{11}^{\alpha} = (\alpha, \alpha + 2)$$

$$c_{11}^{\alpha} \bullet x_{11}^{\alpha} = (2\alpha^2 - 2\alpha\ , -2\alpha^2 - 2\alpha + 4) \text{ ---------- (A)}$$

In the similar way we get,

$$c_{24}^{\alpha} \bullet x_{24}^{\alpha} = (4\alpha^2 + 6\alpha + 1\ , 4\alpha^2 - 20\alpha + 25) \text{ ------ (B)}$$

$$c_{31}^{\alpha} \bullet x_{31}^{\alpha} = (2\alpha^2 + 4\alpha + 2\ , 2\alpha^2 - 12\alpha + 18) \text{ -------(C)}$$

$$c_{32}^{\alpha} \bullet x_{32}^{\alpha} = (4\alpha^2 + 8\alpha\ , 4\alpha^2 - 24\alpha + 32) \text{ ---------- -(D)}$$

$$c_{33}^{\alpha} \bullet x_{33}^{\alpha} = (4\alpha^2 + 9\alpha + 4\ , 4\alpha^2 - 26\alpha + 40) \text{ ------ (E)}$$

$$c_{43}^{\alpha} \bullet x_{43}^{\alpha} = (6\alpha^2 + 11\alpha + 4\ , 6\alpha^2 - 35\alpha + 50) \text{ ------ (F)}$$

$$\sum_{j,i} Z_{ij} = (22\alpha^2 + 36\alpha + 11, 18\alpha^2 - 119\alpha + 169) \text{ --- (G)}$$

Solving the equation (G), we get

$$\mu_{.\cos t.z(X)} = \begin{cases} \frac{-36 + \sqrt{36^2 - 88(11 - x_1)}}{44}, & 11 \le x_1 \le 68 \\ \frac{119 - \sqrt{119^2 - 72(169 - x_2}}{36}, & 68 \le x_2 \le 169 \end{cases}$$

from the membership function of the optimal solution; it can be find the grade of the fuzzy transportation cost which lies between

α	$[\underline{z_{ij}}, z_{ij}, \overline{z_{ij}}]$	$\sum_{i,j} z_{ij}$	α	$[\underline{z_{ij}}, z_{ij}, \overline{z_{ij}}]$	$\sum_{i,j} z_{ij}$
0	[11,68, 169]	79	.5	[34.5,68,114]	71.1
.1	[14.8,68,157.2]	77	.6	[40.5,68,104]	70.1
.2	[19, 68, 145.9]	75.2	.7	[46.9,68,94.5]	69.3
.3	[23.7, 68, 134.9]	73.6	.8	[53.8,68,85.3]	68.7
.4	[28.9,68,124.2]	72.2	.9	[61.2,68,76.4]	68.4

Conclusion

In this paper, we have obtained an optimal solution for fuzzy transportation problem of minimal cost using fuzzy triangular membership function. The new arithmetic operations on triangular fuzzy numbers are employed to get the fuzzy optimal solutions. The optimal solutions obtained using our method can also be verified in the triangular membership function. This would be a new attempt in solving the transportation problem in fuzzy triangular numbers. Anticipating the valuable comments and suggestions.

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