# A New Technique for Relating Post-Collision to Pre-Collision Speeds in Automobile Accidents

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**Abstract - Many car accidents occur each day around the world and accident reconstruction methods are used to determine each vehicle's speed at the time of accident. However, the accuracy of such methods depends highly on the nature of the available information and the skill of the accident analyst. This paper describes a new technique that relates the post-collision variables to pre-collision variables in car accidents. This technique is based on using the laws of engineering dynamics and strength of materials. Real-life case studies were investigated to compare the calculated variables to the real observed variables.**

***Keywords-component; Collision speeds; accident reconstruction; simulation; collision; impact.***

## I. INTRODUCTION

A collision or impact between two or more bodies is a short duration event that includes changes in the dynamic and material properties of the bodies in impact due to the impulsive forces and change in momentum. This principle is useful for solving problems that involve velocity, force, and time. It can be applied to applications involving both angular and linear motion. It should be noted that most real life impact cases are considered as an oblique. Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.

It is essential to study these dynamic changes from the impact in order to simulate the impact scenario and solve the inverse problem by relating the Post-Collision velocities to Pre-Collision velocities.

Solving this inverse problem for two bodies in impact can be achieved by calculating the pre-collision velocity of each car by given only the post-collision data.

## II. CLASSICAL COLLISION THEORY

If two rigid planner bodies collide and remain in contact for a period of time $\Delta t$ , then for either rigid body,[1-2]

$$\lim_{\Delta t \to o} \int \overline{F}(t)\, dt = \hat{F} = m\,(\overline{v}_2 - \overline{v}_1) \tag{1}$$

where;

$\overline{v}_1$ and $\overline{v}_2$ are the velocities of the mass center of either rigid body before and after impact.

*m*: is the mass of either one of the bodies.

$\overline{F}$ : is the impulsive force which assumed to be the average force acting during the contact.

The angular momentum equation relating the behavior before and after the impact is given by one of the following equations;

$$\int \overline{M}_s\, dt = \hat{M}_p = \overline{H}_{p2} - \overline{H}_{p1} \tag{2}$$

$$\int \overline{M}_s\, dt = \hat{M}_s = \overline{H}_{s2} - \overline{H}_{s1} + \int (\overline{S} + M\,\overline{R})\, dt \tag{3}$$

where;

$\overline{H}_{p1}$ and $\overline{H}_{p2}$ are the angular moments about a point P before and after the collision, respectively.

$\overline{H}_{s1}$ and $\overline{H}_{s2}$ are the angular moments about any point S in the plane before and after the collision, respectively.

$\dot{\overline{R}}$ is the mass center's velocities of the body.

$\dot{\overline{S}}$ is the velocity of point S.

If two rigid bodies of masses $m_1$ and $m_2$ are in impact and their linear and angular velocities change according to the nature of impact, then the analysis will include six unknowns to be determined by six equations relating the post-collision and the pre-collision variables. The unknowns are:

$\overline{v}_{x1}$ and $\overline{v}_{y1}$: the components of the linear pre-collision velocity of body $m_1$.

$\overline{v}_{x2}$ and $\overline{v}_{y2}$: the components of the linear pre-collision velocity of body $m_2$.

$\overline{w}_1$ and $\overline{w}_2$: the angular velocities of the two bodies with only one component in the z direction normal to the x-y plane.

If the two rigid bodies are considered together as one system, then the impact may be considered as an internal effect and therefore the linear momentum at the system center of mass is conserved. Thus Eq. 1 becomes:

$$m_1 \bar{v}_1 + m_2 \bar{v}_2 = m_1 \bar{v}_1' + m_2 \bar{v}_2' \quad (4)$$

which has two components in the x and y directions;

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}' \quad (5)$$

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}' \quad (6)$$

Applying Eq. 3 to the point of contact *Q*, the equation of conservation of the angular momentum about point *Q* can be expressed as;

$$\bar{H}_Q' - \bar{H}_Q + \int_t^{t'} (\dot{\bar{Q}} \times M \dot{R})\, dt = 0 \quad (7)$$

Assuming that the phenomenon is instantaneous, thus; $t = t'$ and $\int_t^{t'} (\dot{\bar{Q}} \times M \dot{R})\, dt = 0$

then, Eq. 7 becomes $\bar{H}_Q = \bar{H}_Q'$ or;

$$\bar{R}_1 \times m_1 \bar{v}_1 + I_1 \bar{w}_1 = \bar{R}_1' \times m_1 \bar{v}_1' + I_1 \bar{w}_1' \quad (8)$$

$$\bar{R}_2 \times m_2 \bar{v}_2 + I_2 \bar{w}_2 = \bar{R}_2' \times m_2 \bar{v}_2' + I_2 \bar{w}_2' \quad (9)$$

Where;

$I_1$ and $I_2$ are the mass moments of inertia about each body's mass centre.

Equations (8) and (9) along with the two linear momentum equations give a total of four equations. The remaining two equations are established based on the type of friction between contacting points.[2] [3] [4]

Assuming coulomb's friction model, the following possibilities exist:

1- Areas of contact are smooth, then the coefficient of friction between contacting points is $\mu = 0$ . This means that only normal forces, but no tangential forces exist at the points of contact. Thus the following scalar equation is obtained:

$$m\bar{v} \times \bar{n} = m\bar{v}^{-1} \times \bar{n} \quad (10)$$

2- Areas of contact are rough but they do not slip, $\mu > 0$. In this case the tangential velocities of both vehicles at the time of separation are identical. Thus;

$$\bar{V}_{Q1}' \times \bar{n} = \bar{V}_{Q2}' \times \bar{n} \quad (11)$$

3- Areas of contact are rough and they slip, $\mu > 0$. The relation in this case based on the coefficient of friction may be given as:

$$\mu\, (\hat{F}.\bar{n}) = \left|\hat{F} \times \bar{n}\right| \quad (12)$$

or;

$$\mu m_1 \left(\bar{V}_1' - \bar{V}_1\right).\bar{n} = m_1 \left|\left(\bar{V}_1' - \bar{V}_1\right) \times \bar{n}\right| \quad (13)$$

$$\mu m_2 \left(\bar{V}_2' - \bar{V}_2\right).\bar{n} = m_2 \left|\left(\bar{V}_2' - \bar{V}_2\right) \times \bar{n}\right| \quad (14)$$

Therefore, the two linear momentum equations, the two angular momentum equations and the two equations of contact form a total of six equations. These velocities are solved simultaneously to determine the pre-collision velocities. However, these velocities are subjected to the assumptions made which do not well suit the automobile collision problem.

## III. COLLISION OF TWO VEHICLES

The collision of rigid bodies was analyzed in an ideal form in the preceding section. In this section, collision of two vehicles will be analyzed in a more practical form. Consider two vehicles traveling in two dimensional plane x and y. At the time t the two vehicles collide and stay in contact as one system for a short period of time *Δt*.[2][4]

The kinetic energy of the system at the time of contact can be expressed as:

$$KE = KE_1 + KE_2 \quad (15)$$

Where,

$$KE_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 w_1^2 \quad (16)$$

And

$$KE_2 = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 w_2^2 \quad (17)$$

$v_1$ and $v_2$ are the pre-collision speeds of the two vehicles

$w_1$ and $w_2$ are the angular speeds of the two vehicles at the time of contact.

Thus,

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_1 w_1^2 + \frac{1}{2} I_2 w_2^2 \quad (18)$$

The last two terms are the pre-collision energy of rotation, the are usually small in vehicle collision when compared to the first two terms, thus it can be assumed that;

$$KE=\frac{1}{2}m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (19)$$

A resultant force F appears on the contacting surfaces instead of the points of contact. The kinetic energy is reduced by the work done by the collision force F (*CE*) and the work done by the two vehicle's tires on the road (*TW*).

Thus,

$$\Delta KE=CE + Tw \quad (20)$$

Where *Δ KE* is the reduction in kinetic energy.

$$Tw=\sum_{i=1}^{4\,wheels}(friction\,force)\,(skid\,dist.) \quad (21)$$

$$Tw=\sum_{i-1}^{4}(\mu(m)_i\, g)(SD)_i \quad (22)$$

Calculating the work done by the tires of each vehicle separately,

$$Tw_1=(\mu\, g\, m_1)(SD)_1 \quad (23)$$

$$Tw_2=(\mu\, g\, m_2)(SD)_2 \quad (24)$$

where, $\mu$ is the coefficient of friction between the tires and the road. $\mu$ is usually taken as 0.6-0.7 for dry pavement surfaces.[5]

thus,

$$\Delta KE=CE+\mu\left[m_1\,(SD_1)+m_2\,(SD_2)\right] \quad (25)$$

The crush energy is usually much higher than the tire work. The kinetic energy of the system at time t' is similarly expressed as:

$$KE'=KE_1^{'} + KE_2^{'}$$

$$=\frac{1}{2}m_1\, v'^2_1+\frac{1}{2}I_1\, w'^2_1+\frac{1}{2}m_2\, v'^2_2+\frac{1}{2}I_2\, w'^2_2 \quad (26)$$

The change in kinetic energy during contact may now be expressed as

$$\Delta KE=KE-KE' \quad (27)$$

By substitution and using equations (18), (23), and (24), the following equation can be achieved;

$$CE+\mu\left(m_1\,.SD_1+m_2\,.SD_2\right)=\frac{1}{2}m_1\, v_1^2+\frac{1}{2}m_2\, v_2^2$$

$$-\left(\frac{1}{2}m_1\, v'^2_1+\frac{1}{2}I_1\, w'^2_1+\frac{1}{2}m_2\, v'^2_2+\frac{1}{2}I_2\, w'^2_2\right) \quad (28)$$

The above equation relates post collision speeds to pre-collision speeds and can be tested on reconstructed collision cases.

## IV. KINETIC ENERGY LOSS DURING CONTACT

A large amount of the change in the kinetic energy of the two vehicles during the contact is caused by the damage that takes place in the two vehicles [5]. This amount is very difficult to evaluate accurately. However an acceptable estimation can be done by using the theory of plates [6]. The final deformation of a vehicle body is obtained in two phases: in the first phase, the deformation is elastic and the energy involved is given by an expression in the form of $\frac{1}{2}KX_e^2$, where K is the stiffness of the deformed region and $X_e$ is the deflection at the limit of elasticity. In the second phase, the deformation is plastic during which the energy involved is expressed by $F_Y\left(X_f-X_Y\right)$ where $X_f$ is the final deflection, $X_y$ is the deflection at the yielding point and $F_Y$ is the force at the yielding point. Knowing that $F_Y \simeq KX_e$ and $X_e \approx X_y$, then the total energy involved in the deformation process is given by $\frac{1}{2}KX_y^2+F_y\left(X_f-X_y\right)$ and can be approximated by $F_Y\, X_f$.[7][8]

The final deflection $X_f$ can be obtained by direct measurement while $F_Y$ is to be estimated using the theory of plates. The deformed area can be considered as a flat plate having a rectangular shape of length (a) and width (b), and having all edges fixed under a load (q) that is uniformly distributed over the entire plate. At the center, the stress can be expressed as;

$$\sigma=\frac{\beta_2 q b^2}{t^2} \quad (29)$$

Where, $\beta_2$ is a factor depending on the ratio of a/b. It varies between 0.1386 for a/b = 1 and 0.2472 for $\frac{a}{b}=2$. It is considered as equal to 0.25 for a/b > 2. [8]

This expression of $\sigma$ is valid in the elastic range. It will be used at the limit of range to obtain the uniform load causing the yielding. This load will be assumed to remain constant during the whole process of plastic deformation. This assumption is acceptable from the physical point of view.

The value of $\sigma_y$, the yielding stress, depends on the steel used. It is not given by the manufacturer in general. It is likely between 150 and 250. Therefore, if an average value of 200 Mpa is adopted, then;

$$q = \frac{\sigma_y t^2}{\beta_2 b^2} \tag{30}$$

Besides the energy of deformation, a large amount of kinetic energy is dissipated into heat, but this amount cannot be evaluated.

## V. APPLICATION ON CASE STUDIES

In the application, it is logical to neglect the kinetic energy due to the rotation with respect to that resulting from the linear motion of the vehicle. Two cases are considered as detailed below.

### A. Case One

The field study case is related to an accident between a Peugeot 604 1986 (car 1) and a Chevrolet Spectrum 1985 (car 2). The observed post collision data are giving in Table 1.

Applying the above equations on the collision case in Table 1 and taking μ value as 0.6 will lead to the following calculations just after collision:

$$\frac{1}{2} m_1 v_1^{'2} - \mu m_1 g d_1 = 0 + v'_1 = \sqrt{2\mu g d_1} = 16.093\, m/s$$

$$\frac{1}{2} m_2 v_2^{'2} - \mu m_2 g d_2 = 0 + v'_2 = \sqrt{2\mu g d_2} = 14.797\, m/s$$

$$q_1 = \frac{\sigma_y t_1^2}{\beta_2 b_1^2} \text{ with } \beta_2 = 0.25,\ q_1 = 0.01653 \text{ N/mm}^2$$

$$q_2 = \frac{\sigma_y t_2^2}{\beta_2 b_2^2} \text{ with } \beta_2 = 0.2286,\ q_2 = 0.01116 \text{ N/mm}^2$$

TABLE 1 POST COLLISION DATA FOR THE CRASH IN CASE 1

| | **Car 1** | **Car 2** |
|---|---|---|
| **Mass** | $m_1$ = 1500 kg | $m_2$ = 1200 kg |
| **Thickness of body** | $t_1$ = 2.5 mm | $t_2$ = 2.5 mm |
| **The damaged area** | $a_1$ = 147 cm<br>$b_1$ =55 cm | $a_2$ = 112 cm ,<br>$b_2$ = 70 cm |
| **Average deformation** | 20 cm | 18 cm |
| **Length of skid marks** | $d_1$ = 22 m | $d_2$ = 18.6 m |

Then, during collision, the impulsive forces are:

$F_1 = q_1 A_1 = 13364.5\ N$
$F_2 = q_2 A_2 = 8749.4\ N$

The corresponding energies of deformation are

$W_1 = F_1 X_1 = 2673\ J$
$W_2 = F_2 X_2 = 1575\ J$

Furthermore,

$$m_1 V_1 - \sum F_1\, \Delta t = m_1 V_1^{'}$$

$$m_2 V_2 - \sum F_2\, \Delta t = m_2 V_2^{'}$$

Δt may be taken as 0.5 s and $\sum F$ include the impulsive force and the friction force. Then,

$$V_1 = V_1^{'} + \left(\frac{F_1}{m_1} + \mu g\right)\Delta t = 23.49\, m/s = 84.57\, km/h$$

$$V_2 = V_2^{'} + \left(\frac{F_2}{m^2} + \mu g\right)\Delta t = 21.39\, m/s = 77.00\, km/h$$

These values give only an idea about the velocities before collision. The collision time Δt is very difficult to estimate accurately and it plays an important role in the determination of velocities.

The actual velocities are higher than the values obtained theoretically since the kinetic energies dissipated into heat are not taken into account.

### B. Case Two

The second field study case is related to an accident between a Toyota Corolla, 1981 (car 1) and a Nissan, GT, 1983 (car 2). The observed post collision data are giving in Table 2.

TABLE 2 POST COLLISION DATA FOR THE CRASH IN CASE 2

| **Car 1** | **Car 2** |
|---|---|
| $m_1$ = 1370 kg | $m_2$ = 1205 kg |
| $t_1$ = 2.7 mm | $t_2$ = 2.6 mm |
| $d_1$= 9 m | $d_2$ = 25 m |
| $a_1$ = 60 cm, $b_1$ = 44 cm | $a_2$ = 32 cm , $b_2$ = 28 cm |
| $x_1$ = 8.4 cm | $x_2$ = 16 cm |
| $\beta_{21}$ = 0.1944 | $\beta_{22}$ = 0.1390 |

Just after collision, for the two vehicles:

$$V_1^{'} = \sqrt{2\mu g d_1} = 10.293\, m$$
$$V_2^{'} = \sqrt{2\mu g d_2} = 17.155\, m$$

$q_1$ = 0.03874 N/mm$^2$
$q_2$ = 0.12406 N/mm

Then, during collision, the impulsive forces acting are:

$F_1$ = 10227.36 N
$F_2$ = 11115.78 N

Then, the velocities of the two cars before collision can be calculated as;

$v_1$ = 16.97 m/s = 61 km/h
$v_2$ = 24.71 m/s = 89 km/h

And as discussed above, the actual velocities are higher than the values obtained theoretically since the kinetic energies dissipated into heat are not taken into account.

## VI. CONCLUSION

It was shown that the second law of dynamics together with plate theory can lead to an estimation of vehicle velocities before and after collision. The study is based upon a certain number of realistic assumptions. The only drawback is that the amount of kinetic energy dissipated into heat during the collision was neglected. A more accurate analysis could be conducted if experimental results concerning the fraction of kinetic energy converted into heat were known.

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